**Weiner Process**

The Wiener process forms the basis, for my purposes, in Stochastic Calculus. So I guess we’ll start here.

**Vector Wiener Process**

Let’s generalize to a vector Weiner process, d**W**(t). It’s pdf should be something like:



Actually, wouldn’t it be more like,



The old formulas follow:



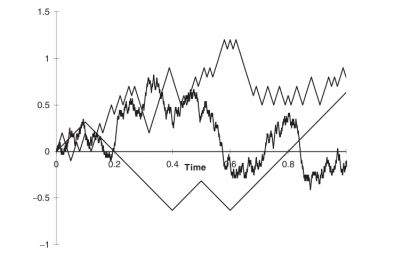
We can write this another way, defining w = dW/dt,



From this we can define the Wiener process whose distribution would follow from the definition.



Because the Wiener process can be expressed as the sum of n = anything, independent and identically distributed variables, it is considered and **infinitely divisible process**. It looks something like this:



And relatedly ΔW = W(a+Δt) – W(a), has distribution



And correlations between different segments would be the following, where Δtoverlap = the overlap of the time intervals Δt and Δt’.



**Integral Functions of Wiener process**

Taking a look below, at the ‘differential functions of W’ section, we have:



Integrating w/r to t, I think we may conclude that:



**Example**

Let’s consider the function F(W1,W2) = W1W2. Then,



So I guess you could say:



But is there any way to solve for ∫W1dW2 by itself? For instance, can we say something like:



?? Let’s consider the expectation of these integral, in the Stratonovich form. So we have:



Can this be proven? So, uh,



Gonna separate out parts that correlate with ΔW2:



Well, average is zero still souer that’s good.



The first term will go to zero because only j = i will have a non-zero net value. But then the (dt)2 will bring it down to zero. So then we’re left with the other three terms. Basically we need cross terms to be non-zero for the equality not to hold. So let’s just look for non i = j terms.



Let’s look at the first one:



This is just going to go to zero.



Well this also should go to zero. And the last:



note tbelow,i includes ti-1/2, and tabove,i includes ti+1. For the first set of four, if i < j, then ε2(tj) will be unmatched. Same if i > j. What about last term? if i > j, then everything will vanish because there will be a lowest unmatched ε2(tj) or ε2(tj+1/2). Same with i < j. What about the middles though? Take the 3rd. Seems that if i < j, there is likelihood of this term surviving. So let’s work it out:



So the two brackets on the left must match one of the four products in the rightmost bracket. Let’s set i = 10, and j = 20. Then:



And the surviving terms are:



So this seems to be viable (and bad). And the whole thing seems to be symmetric w/r to interchange of i and j, so the other guy shouldn’t cancel this one out. So it appears this identity does not hold.

**Example 2**

Or how about just a function of t? Consider,



Let’s call it **F**(t). Is it just a weighted sum of Gaussians and is therefore a Gaussian itself? We can find the mean and variance,



What are higher moments?



Let’s look at:



Using,



and path integral integration – see folder, we can see that this is just:



So,



which is exactly how a normally distributed variable would behave. So I think it is the case that **F**(t) follows a multivariate normal distribution with mean 0, and covariance **D**. So,



I’m pretty sure our boxed formula is correct. Note the use of ~, rather than =, which means that the two have the same probability distribution. But not that they are identical per se´. This is in the same sense that two random variables X and Y can have the same probability distribution but be independent variables, or identical (completely dependent) variables, or perhaps just somewhat dependent too.

**Differential Functions of Weiner process**

I’m just going to consider derivatives for now. We’ll define:



and then, expanding the numerator about some point within the interval…



Again we replaced WW´ terms with <WW´>, which isn’t technically okay outside integrals, and maybe not always okay even then. But hopefully it is okay. Equating the first and last lines, dividing by tk+1 – tk we appear to have:



We can see that the value of the derivative depends on λ of course. We can write this more succinctly as,



So the λ would govern where, in between the (t,t+dt) interval, the partial derivatives are evaluated. The Ito version evaluates at the left end point, for which λ = 0, and the Stratonovich version evaluates at the midpoint, for which λ = ½, and the last term vanishes. What would a product rule look like? This also follows quite simply from the definition of dF/dt itself, since:



So we have:



Another way is to attempt this is (implicit summation):



which doesn’t match, interestingly, accept in the Ito case, and for a symmetric Dij. So doing the differential approach is not especially generally true. And it fails utterly for the Stratonovich derivative, which itself preserves the normal rules of calculus. What about the chain rule? Consider F(W,t) = F(G(W,t)). And again, the fundamental rule will suffice:



We could say:



Again, the Stratonovich version preserves normal calculus of course. But the Ito version adds an extra term.